



# Kriging, indicators, and nonlinear geostatistics

by J. Rivoirard\*, X. Freulon†, C. Demange†, and A. Lécureuil†

## Synopsis

Historically, linear and lognormal krigings were first created to estimate the *in situ* mineral resources of blocks. Nonlinear geostatistics and indicator kriging were subsequently developed to evaluate also the portion recovered when applying a cut-off on selective mining units (SMUs) within blocks. In practice these methods are generally based either on the Gaussian model with a transformation generalizing the lognormal case or on the indicators above cut-offs. Very often the indicator approach is simplified by kriging separately each indicator, and when starting from a continuous variable, a practical advantage of the discretization into classes lies in the easy treatment of a zero effect and of the high values. However, a number of so-called isofactorial models have also been developed for a discrete or continuous variable, where the full cokriging of indicators (i.e. disjunctive kriging) simplifies to the separate kriging of factors. Moreover, these models are equipped with a change of support, allowing a consistent estimation of recoverable resources on SMUs.

Min-Max Autocorrelation Factors (MAF) analysis of the indicators offers a new approach for indicator modelling. In particular the first factor, the one with the highest spatial continuity, can help in choosing the type of model. For example a monotonic experimental first factor can be used directly as the basis of a discrete diffusion model, unless a continuous diffusion model such as the Gaussian model can be used on the original variable. This approach is illustrated on a uranium deposit mined selectively: estimates of recoverable resources by discrete disjunctive kriging and uniform conditioning in a Gaussian model are compared locally to short-term estimates based on two areas densely drilled.

## Keywords

disjunctive kriging, indicator kriging, Min-Max Autocorrelation Factors, recoverable resources, discrete diffusion model.

## Introduction

Linear kriging is a recognized and commonly used technique. It allows estimation of a regionalized variable (represented here by  $Z(x)$  where  $x$  denotes a point-support location) at target points, or estimation of sets of points such as blocks in the case of an additive variable.

Although it was a domain of intense research many years ago, nonlinear geostatistics (Rivoirard, 1994; Chilès and Delfiner, 1999) is still a complex and difficult part of geostatistics. In contrast with linear geostatistics, which considers only linear combinations of the variable, nonlinear

geostatistics deals with transformations of the variable, for example a Gaussian transformation or indicators.

One purpose of nonlinear geostatistics is the estimation of the target variable itself at points or over blocks. Another one is the estimation of a transformation of the variable (Vann and Guibal, 2000). Typically it is used to predict the exceedence of the threshold at points (e.g., the indicator  $\text{Ind}\{Z(x) \geq z\}$ ) or for blocks ( $\text{Ind}\{Z(v) \geq z\}$  where  $v$  denotes the block support). The latter case, which corresponds in particular to the recoverable resources of selective mining units (SMUs) above cutoff  $z$ , is more complex as it cannot be reduced to a simple block averaging and demands a change of support.

In practice today, the two most commonly applied nonlinear approaches are Gaussian-based methods such as uniform conditioning or multiple indicator kriging (IK) (Journel, 1982; Vann *et al.*, 2000). The choice of one technique or the other is often made *a priori*, and the existence of other methods is ignored. In particular, the fact that local estimation is better supported by the transformation of the variable that presents the highest spatial correlation is largely unexploited.

In this paper we propose to use the Min/Max Autocorrelation Factors (MAF) (Switzer and Green, 1984; Desbarats and Dimitrakopoulos, 2000) of indicators as a tool to determine the transformation that yields the highest spatial correlation and to choose the model for nonlinear geostatistics accordingly. This model can then be used to compute local estimates using disjunctive kriging. This approach extends the normal practice of multiple IK as it takes into account cross-correlations between the indicators and includes an appropriate change-of-support model, rather than an arbitrarily chosen one.

\* MINES ParisTech, Centre de Géosciences, Fontainebleau, France.

† AREVA Mines, Paris La Défense, France.

© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.

## Kriging, indicators, and nonlinear geostatistics

The paper is structured as follows. We first present an overview of the main nonlinear models and methods. Then we consider the use of MAF of indicators for the choice of the model. Finally, we present a case study for the local estimation of mineral resources on real data.

### Overview of methods and models

The Gaussian transformed model is very commonly applied, both in nonlinear geostatistics and in conditional simulation. The transformation generalizes the lognormal distribution used at the origin of geostatistics (Krige, 1951, 1952, 1978).

Thanks to a multi-Gaussian hypothesis, the Gaussian model easily provides the conditional distribution (and so the conditional expectation) at target points (Verly, 1983). It is adapted to an original variable which has a continuous distribution and in particular has no zero effect (a high proportion of zeroes can cause a problem for the inversion into Gaussian values). The Gaussian model is also equipped with a change-of-support model. A variant technique is the uniform conditioning (UC) under the Gaussian model (Rivoirard, 1994; Deraisme *et al.*, 2008), which predicts the distribution of SMU values in a panel, conditional on an estimate of this panel (relaxing the constraints on zero effect and stationarity).

In the indicator approach, the variable is either discrete or if continuous is discretized into classes. This can be convenient in case of a zero effect, as well as in case of high values that can be grouped into one class. IK consists of independently kriging the indicators of (cumulated) classes, or in estimating linearly each class with a common variogram. Its advantage relies on its simplicity. It does not, however, exploit the joint structure of indicators nor, when using a common variogram, the common destructure of high grades (Vann *et al.*, 2000). In addition, it is often used with a posterior change of support that does not guarantee consistency (Emery, 2008). Furthermore, post-processing is required to obtain estimates of indicators at all possible cut-offs.

Disjunctive kriging (DK) is the original name for indicator cokriging (Matheron, 1976). The required knowledge of bivariate distributions between points is actually equivalent to the multivariate structure of indicators. DK is possible with both discrete and continuous distributions.

In the case of a mosaic model with independent valuations (Rivoirard, 1984), all structures are the same and DK reduces to independent kriging of each indicator class (this is the model where IK would be optimal at point support).

A number of so-called isofactorial models were developed years ago (refer in particular to the numerous papers by Matheron cited in Chilès and Delfiner, 1999). The factors have no spatial cross-correlation, so that DK is obtained by kriging these separately. Moreover, internally consistent change-of-support models exist for all of these models. An example of such an isofactorial model is the Gaussian model, where bivariate distributions are Gaussian, and factors are the Hermite polynomials of the Gaussian transformed variable. Another example is the gamma model, whose factors are the Laguerre polynomials (Hu, 1988).

Such models are diffusion-type models: intermediate values are met when going from low to high values and *vice-*

*versa*. Moreover, they are based on a defined probability distribution: Gaussian, gamma, etc. The observation of elliptically-shaped scatter plots between points separated by a given distance, typical of bi-Gaussian distributions, for instance, is in practice a good vector for the choice of a Gaussian model, before modelling the variogram of the Gaussian transformed variable.

On the other hand, the indicator approach considers directly structural tools such as simple or cross-variograms. We have already seen the very particular case of the mosaic model. Two isofactorial models are especially interesting here, which correspond to finite discrete distributions and which are not based on a defined probability distribution. This makes them suitable to describe an indicator transformed data-set. One is the discrete diffusion model developed by Matheron (1976) and Lajaunie and Lantuéjoul (1989). Its modelling is based on the experimental determination of its first factor, the one which carries the highest spatial continuity. This should be monotonic with respect to the variable under study (this is a condition for diffusion, representing the transition from low to high values through medium ones). All other factors are derived from this first factor. The other model is the model with indicator residuals (IR) (Rivoirard, 1989). It is not a diffusion-type model, but a hierarchical one, where the first factor is one basic indicator and the other factors are the successive residuals of indicators.

Discrete disjunctive kriging (DDK), based on a discrete isofactorial model, can handle zero-effect or atoms (i.e. 0 or other values observed with a high proportion), extreme values, and includes a change of support. It can be viewed as an extended IK as it overcomes recognized limitations of the IK approach: specifically, it takes into account the cross-correlations between the indicators and includes a consistent change-of-support model. The parameters of these discrete models (discrete diffusion or IR) can be derived from the analysis of MAF computed on indicators, since the MAF of indicators correspond to an experimental version of the factors of an isofactorial model.

### Using MAF of indicators for the choice of model

The fact that the local estimation should be driven by what corresponds to the major spatial continuity in the variable is largely unexploited in many geostatistical approaches. For instance, in a transformed Gaussian model, the Gaussian variable is known to be the transformed variable that has the highest continuity (and the estimation is based on the kriging of the Gaussian variable), but this fact is not used for choosing the model. This is where MAF of indicators (after discretization of the variable) can be useful (as linear combinations of indicators, MAF are the same whether cumulated indicators are used or not). Remember that MAF are multivariate statistics which are for spatial statistics or geostatistics what principal components (PCs) are for statistics: in both cases they are orthogonal linear combinations of the initial variables, but in the case of MAF they are based on spatial continuity instead of statistical variability in the case of PCs. MAF are ranked by decreasing spatial continuity at a chosen separation vector  $h$ , and their cross-correlation at this distance and at distance 0 is zero.

## Kriging, indicators, and nonlinear geostatistics

Modelling MAF assumes that their cross-correlation is zero for other distances, which can be checked on the experimental cross-variograms of the MAF. The absence of cross-correlation over all distances implies that an isofactorial model fits the data. In addition, computing the MAF of indicators provides valuable structural information. In particular the first MAF is the linear combination of indicators that presents the highest spatial continuity. It is a function of the discrete variable under study, and its composition in term of values or classes on the original variable is meaningful.

A first factor which is a stepwise function of the variable (i.e. an indicator) could orientate towards an IR model. With a monotonic first factor, the structure is essentially due to the gradual transition from low to high values. This corresponds to a diffusion-type model, and could support the use of a discrete diffusion model or another diffusion-type model such as a Gaussian or gamma model. MAF that do not correspond to existing models (e.g. a first factor that is not monotonic) would require the development of new models, enabling kriging MAF and possibly including a change of support.

### Application to a uranium mine

In the following section, discrete disjunctive kriging is applied to drill-holes from a uranium project in a horizontal stratabound sediment-hosted deposit: two areas were densely drilled to establish the grade variogram, especially for short distances, and to evaluate long-term resource models. The data-set used to illustrate the methodology contains composites derived from radiometric measurements in vertical drill-holes and has been split in two subsets: (a) holes drilled on a regular 50 m centred grid used to establish long-term models on 25×25×2 m<sup>3</sup> panels (or long-term data-set); (b) holes drilled on a regular 12.5 m centred grid in two 50×50 m areas (or short-term data-set). Basic statistics on 2 m length composites are reported for both data-sets in Table I.

Two long-term resource models have been computed considering 5×5×2 m<sup>3</sup> SMUs, the first one using the uniform conditioning in a Gaussian model (UC); the second using discrete disjunctive kriging (DDK); both are based on the same long-term data-set. Finally, a uranium grade model on 5×5×2 m<sup>3</sup> blocks is computed using composites from the short-term data-set. This model, restricted to the 50×50 m areas, is similar to those used to guide the production and has been considered as the reference.

The application of uniform conditioning follows the classical approach (e.g. Rivoirard 1994); it is not described further and only the application of discrete disjunctive kriging and the comparisons are presented in more detail.

The discrete approach requires a disjunctive coding of the initial variable: a set of thresholds ( $0 \leq z_1 \leq \dots \leq z_i \leq \dots \leq z_N$ )

define indicators of ore classes and a discrete average grade can be calculated as follows:

$$\hat{Z} = \sum_{i=0}^N \hat{z}_i \times (1_{Z \geq z_i} - 1_{Z \geq z_{i+1}}) \quad [1]$$

The histogram of the discrete grade is completely specified by the proportion and the average value of each class (Table II). In case of clustered data, a declustering procedure should be applied to derive the appropriate proportions attributable to each class interval.

The selectivity curves of the raw variable and its discrete counterpart are compared in Figure 1: the mean is preserved (i.e. the metal content) and the selectivity is slightly reduced (-3% of the selectivity index, i.e. the Gini coefficient). The reduction of the variance is higher (-50%) as the values of

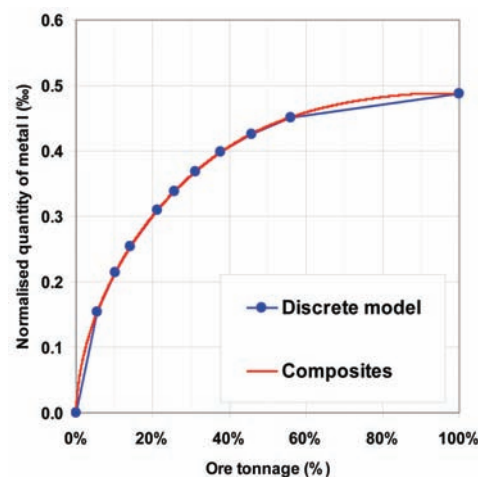


Figure 1—Selectivity curves Q(T) of the continuous grade and its corresponding discrete distribution for 10 classes

Table II

### Parameters of the discrete distribution derived from composite grades

Class index	Limits (‰)	Proportion (%)	Average grade (‰)
0	[0, 0.2 [	43.90%	0.08
1	[0.2, 0.3 [	10.38%	0.25
2	[0.3, 0.4 [	7.91%	0.35
3	[0.4, 0.5 [	6.56%	0.45
4	[0.5, 0.6 [	5.60%	0.55
5	[0.6, 0.7 [	4.52%	0.65
6	[0.7, 0.9 [	6.90%	0.79
7	[0.9, 1.1 [	3.97%	0.99
8	[1.1, 1.5 [	4.77%	1.28
9	[1.5, + [	5.48%	2.80

Table I

### Statistics of uranium grade (‰) from drill-holes (2 m composites)

Data-set	Number	Minimum	Maximum	Mean	Variance	$\sigma/m$
Long-term	9 886	0.00	57.88	0.49	0.90	1.94
Short-term	4 090	0.00	36.17	0.70	1.55	1.77



## Kriging, indicators, and nonlinear geostatistics

the upper tail of the histogram are summarized with a single value. The thresholds were selected between 0.2 and 1.5% for practical considerations. It is not necessary that the thresholds be evenly spaced. They could be defined as those minimizing the reduction of the selectivity index (or the variance) for a given number of classes, but such optimal thresholds were not considered here.

MAF (Switzer and Green, 1984) of the indicators have been computed for different lag values, along the drill-holes and horizontally. Figure 2 reports the experimental values of the first factor as a function of the class index. The first factor is fairly independent of the lag value used to calculate the MAF. This result shows that the isofactorial assumption is reasonable for this application.

The first factor is strictly monotonic, which is a good indication for the use of a discrete diffusion model. Details on this model and its implementation can be found in Lajaunie and Lantuéjoul (1989). The development of the model is sophisticated and out of the scope of the present paper. In contrast, the model is very easy to use for a practitioner having access to dedicated software. The reason is that the model is entirely defined by the marginal distribution of the discrete variable (i.e. the proportions of the N+1 classes) and by the first factor. The first factor is the function of the discrete variable which presents the highest spatial correlation: it is directly derived from data and coincides with the first MAF of the indicators. It has to be monotonic. All the other factors of the model are automatically deduced from this first factor. Figure 3 compares the experimental factors derived from the MAF analysis of the indicators and the factors of the fitted diffusion model. As we can see, the diffusion model describes accurately the experimental factors.

Experimental variograms of MAF are reported in Figure 4. They have been computed along the vertical drill-holes and horizontally (no anisotropy was observed horizontally). They show that the spatial correlation of factors decreases with their rank (this is consistent with their definition) and are significant only for the first three factors. It has been checked that the cross variograms show no significant spatial structure between factors.

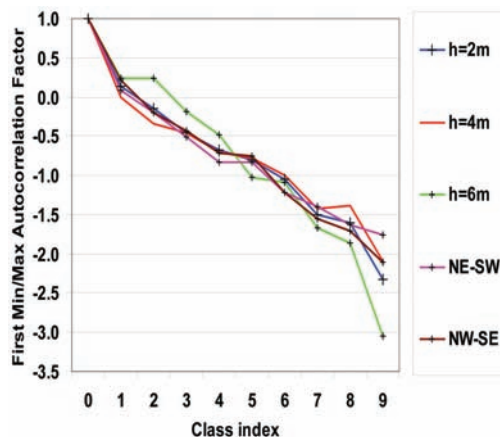


Figure 2—First Min/Max Autocorrelation Factor derived from indicators. Experimental factors have been computed for different reference lags ( $h = 2, 4,$  and  $6$  m along lines,  $35$  m horizontally in SE-NW and SW-NE directions)

In the case of the discrete diffusion model, the covariance functions of the factors ( $\chi_i$ ) are derived from the covariance of the first factor, according to the following formula:

$$\text{Cov}(\chi_i(x), \chi_i(x+h)) = \rho_i(h) = \rho^{\lambda_i}(h) \quad [2]$$

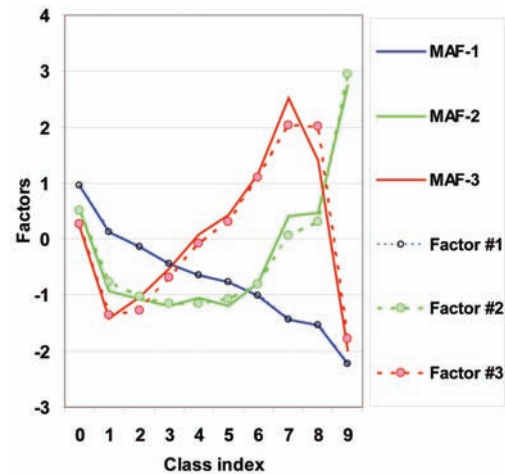


Figure 3—Comparison of Min/Max Autocorrelation Factors ( $\Delta=2$  m) of the indicators and the factors of the discrete diffusion model. This is determined from the 1st MAF and the marginal distribution

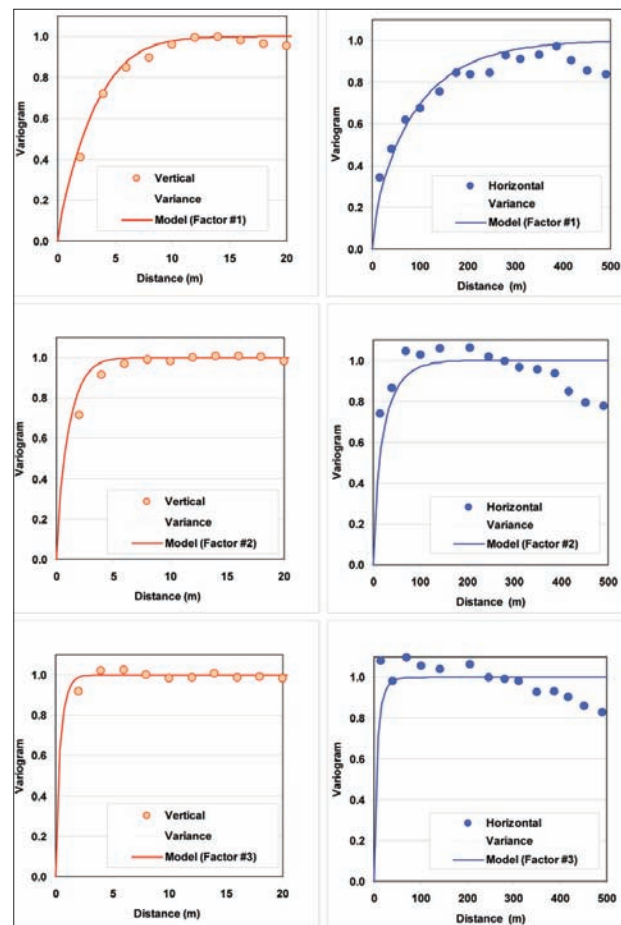


Figure 4—Experimental variograms of the diffusive factors and models. Factor models are derived from 1st factor model

## Kriging, indicators, and nonlinear geostatistics

In this formula the power coefficients ( $1 < \lambda_i < \lambda_{i+1}$  for  $i > 1$ ) are characteristics of the diffusion model, which are computed automatically from the marginal distribution and the first factor.

The covariance of the discrete grade can be derived also from the covariance of the first factor:

$$C(h) = Cov(\hat{Z}(x), \hat{Z}(x+h)) = \sum_{i=1}^N c_i^2 \times \rho^{\lambda_i}(h) \quad [3]$$

where the  $c_i$  are the linear coefficients to compute the discrete grade (with mean  $m$ ) from the factors:

$$\hat{Z}(x) = m + \sum_{i=1}^N c_i \times \chi_i(x) \quad [4]$$

The variogram models fitted to the experimental variograms are reported in Figures 4 and 5: all models are derived from the covariance model of the first factor; this covariance is  $\exp\{-\tau(h)\}$  where  $\tau$  is the variogram:

$$\tau = \text{power } (\alpha=0.7, a_{xy}=100\text{m}, a_z=7\text{m}) + 1.5 \text{ power } (\alpha=1.5, a_{xy}=400\text{m}, a_z=7\text{m})$$

This model can be used to compute the cokriging of any linear combination of the factors, thus any function of the discrete grade. An application is the disjunctive kriging estimate of the grade on SMU or panels, i.e. the short-term model, or the *in situ* resources of the long-term model.

To compute local estimates of the part of *in situ* resources that can be recovered by selective mining, a change-of-support model is necessary. For the discrete diffusion model (Matheron 1984; Lajaunie and Lantuéjoul, 1989), the change of support is specified by a coefficient  $s$  ( $s > 0$ ) derived from the variance of the grade for the block support (calculated from the variogram of the grade):

$$Var(\hat{Z}(v)) = \sum_{n=1}^N c_n^2 \times \left( \frac{1}{1 + \lambda_n} \right)^s \quad s \geq 0 \quad [5]$$

The variance of the discrete grade computed on the composites is 0.42. The variance of the grade for the block  $5 \times 5 \times 2 \text{ m}^3$  is derived from the sill of the regularized model; its value is 0.32. The coefficient of the change of support computed from Equation [5] is  $s = 0.33$ .

Once this coefficient  $s$  has been computed, the covariance of the first factor of the blocks,  $\rho_v$ , is deduced from the regularized model of the discrete grade of the composites:

$$Cov\{\hat{Z}(v), \hat{Z}(v')\} = \frac{1}{|v|^2} \int_{v \times v'} C(x-y) dx dy = \quad [6]$$

$$\sum_{n=1}^N c_n^2 \times \left( \frac{1}{1 + \lambda_n} \right)^s \times \rho_v^{\lambda_n}(v, v')$$

Similarly to the model for composites, the model for the first factor of the blocks is  $\exp\{-\tau_v(h)\}$  where  $\tau_v$  is the variogram :

$$\tau_v = 0.5 \exp(a_{xy}=20\text{m}, a_z=5\text{m}) + 1.5 \text{ power } (\alpha=1.1, a_{xy}=85\text{m}, a_z=5.5\text{m})$$

The simple and cross-covariance functions between point and block factors are then derived from the Cartier relation

(Chilès and Delfiner, 1999). They are used to compute the kriging of any linear combinations of the block factors, in particular the disjunctive kriging of the metal and ore above a cut-off.

On the two areas densely drilled, the selectivity curves giving the metal vs. the tonnage at a given cut-off,  $Q(T)$ , for the Gaussian uniform conditioning and the discrete disjunctive kriging are reported in Figure 6. These estimates are based on the composites of the 50 m centered grid (i.e. the long-term data-set). They are compared with the selectivity curve of the short-term model computed using the composites of the 12.5 m centered grid (i.e. the short-term data-set).

Table III gives the comparison for cut-off 0.3‰ eU. The experimental selectivity curves of the short-term and long-term data-sets are reported also in Figure 6: there is a significant difference between the average grade of the long-term data-set, 0.5‰ eU, and the average grade of the short-term data-set, 0.7‰ eU. The local estimation of the recoverable resources fits, for both methods, the local experimental mean and predicts the selectivity deduced from the short-term model.

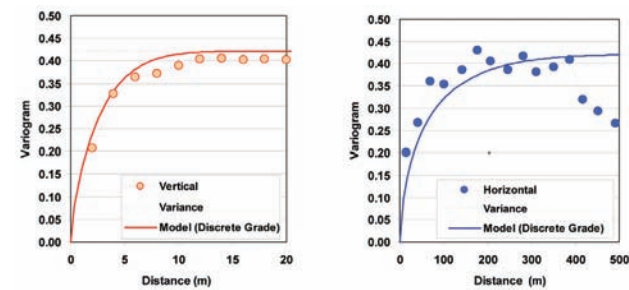


Figure 5—Experimental variograms of the discrete grade and models. The model of variogram is derived from the variogram of the 1st factor model

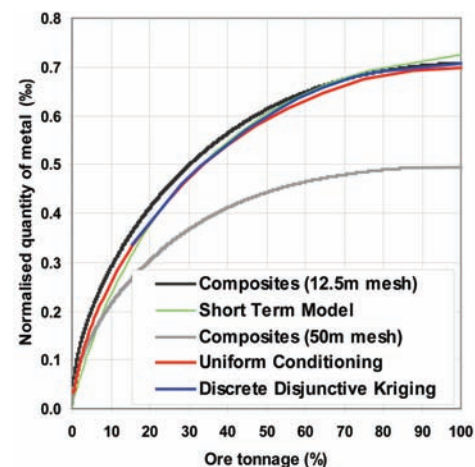


Figure 6—Selectivity curves  $Q(T)$  for long-term models of recoverable resources (uniform conditioning and discrete disjunctive kriging for  $SMU=5 \times 5 \times 2 \text{ m}^3$ ) and short-term model

## Kriging, indicators, and nonlinear geostatistics

Table III

### Comparison of long-term models and short-term model at 0.3‰ cutoff

zc = 0.3‰	Quantity of metal (%)	Ore tonnage (%)	Grade (%)
UC vs. short-term model	-4%	-2%	-1%
DDK vs. short-term model	-2%	-2%	0%
DDK vs. UC	1%	0%	1%

In the present application, two different nonlinear techniques, UC and DDK, give similar results. Both techniques are based on a diffusive model, a continuous one for UC and a discrete one for DDK.

### Conclusion

Nonlinear geostatistics can be viewed as a research technique for the transformation of the variable leading to the highest spatial continuity and being consequently the most appropriate to drive local estimations. One approach to consider is the analysis of the MAF of indicators. This novel point of view makes a direct link between the elementary indicator approach and the more sophisticated models of nonlinear geostatistics. In the present application, the MAF of indicators can be seen to support the choice of a diffusion model.

MAF of indicators analysis also appears as a structural tool on which a methodology can be developed to estimate recoverable mineral resources using DDK. Practical applications may reveal the need to develop new models for MAF of indicators including their change of support. Other subjects of research concern the relaxation of stationarity and the extension to multivariate situations.

### Acknowledgements

The authors thank the management of AREVA Mines for permission to publish the paper. Christian Lajaunie, Karilyn Farmer, and reviewers of SAIMM are gratefully acknowledged.

### References

- CHILÈS, J.-P. and DELFINER, P. 1999. Geostatistics, modeling spatial uncertainty. John Wiley & Sons, New York.
- DERAISME, J., RIVOIRARD, J., and CARRASCO, P. 2008. Multivariate uniform conditioning and block simulations with discrete Gaussian model: application to Chuquicamata deposit, *VIII International Geostatistics Congress, Geostats 2008*. Ortiz, J.M. and Emery, X. (eds.). pp. 69–78.
- DESBARATS, J. and DIMITRAKOPOULOS, R. 2000. Geostatistical simulation of regionalized pore-size distributions using min/max autocorrelation factors. *Mathematical Geology*, vol. 32, no. 8. pp. 919–942.
- EMERY, X. 2008. Change of support for estimating local block grade distributions. *Mathematical Geosciences*, vol. 40. pp. 671–688.

HU, L.Y. 1988. Mise en œuvre du modèle gamma pour l'estimation des distributions spatiales. Doctoral thesis, E.N.S. des Mines de Paris.

JOURNEL, A.G. 1982. The indicator approach to estimation of spatial distributions. *Proceedings of APCOM 17*. Port. City Press, New York. pp. 793–806.

KRIGE, D.G. 1951. A statistical approach to some basic mine valuation problems on the Witwatersrand. *Journal of the Chemical, Metallurgical and Mining Society of South Africa*. Dec. 1951. pp. 119–139.

KRIGE, D.G. 1952. A statistical analysis of some of the borehole values in the Orange Free State goldfield. *Journal of the Chemical, Metallurgical and Mining Society of South Africa*. Sep. 1952. pp. 47–64.

KRIGE, D.G. 1978. Lognormal-de Wijsian geostatistics for ore valuation. Monograph Series, Geostatistics.1. *South African Institute of Mining and Metallurgy*, Johannesburg.

LAJAUNIE, CH. AND LANTUÉJOL, CH. 1989. Setting up the general methodology for discrete isofactorial models. Geostatistics. Armstrong, M. (ed.). Kluwer Academic Publishers, Dordrecht. vol. 1, pp. 323–334.

MATHERON, G. 1976. A simple substitute for conditional expectation: the disjunctive kriging. *Advanced Geostatistics in the Mining Industry*. Guarascio, M., David, M., and Huijbregts, C. (eds.). Reidel, Dordrecht, Holland. pp. 221–236.

MATHERON, G. 1984. Une méthodologie générale pour les modèles isofactoriels discrets. Sciences de la Terre, Annales de l'Ecole Supérieure de Géologie appliquée et de prospection minière, Nancy. vol. 21, pp. 1–78.

RIVOIRARD, J. 1989. Models with orthogonal indicator residuals. Geostatistics. Armstrong, M. (ed.), Kluwer Academic Publishers, Dordrecht, vol. 1, pp. 91–107.

RIVOIRARD, J. 1994. Introduction to Disjunctive Kriging and Nonlinear Geostatistics. Clarendon, Oxford.

SWITZER, P. and GREEN, A.A. 1984. Min/max autocorrelation factors for multivariate spatial imagery. *Technical report no. 6*, Department of Statistics, Stanford University.

VANN, J. and GUIBAL, D. 2000. Beyond ordinary kriging: an overview of nonlinear estimation. *Mineral Resource and Ore Reserve Estimation: the AusIMM Guide to Good Practice*. Australasian Institute of Mining and Metallurgy, Melbourne. pp. 249–256.

VANN, J., GUIBAL, D., and HARLEY, M. 2000. Multiple indicator kriging: is it suited to my deposit? *4th International Mining Geology Conference*. Australasian Institute of Mining and Metallurgy, Melbourne. pp. 9–17.

VERLY, G. 1983. The multigaussian approach and its applications to the estimation of local reserves. *Mathematical Geology*, vol. 15, no. 2. pp. 259–286. ◆