Grinding circuit throughput optimisation through first-principles digital twin modelling

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The global mining industry is increasingly leveraging digital technologies to address persistent challenges in productivity, resource efficiency, and process stability. Grinding circuits, as energy-intensive and highly variable systems, represent a key opportunity for optimisation through real-time, holistic model-based decision support. Traditional operation methods, which are characterised by manual monitoring, and unit-based control strategies, often result in suboptimal throughput and inconsistent product quality. In response, a digital optimisation framework was developed for a gold processing plant in West Africa. The grinding circuit included a SAG mill, ball mill, trommel, pump boxes, and hydrocyclones. A hybrid digital twin framework, combining first-principles and empirical models, was calibrated using operational survey data collected in early 2024. The model was used to maximise hydrocyclone overflow throughput while satisfying process constraints such as particle size, power draw, and hydrocyclone pressure. Three optimisation scenarios are explored, identifying optimal control setpoints yielded 3% throughput gain; suggesting distinct configuration parameters achieved 7.6%; incorporating an additional hydrocyclone reached 15% improvement over the baseline. This paper validates the use of calibrated physics-based digital twins for realising substantial, verifiable performance enhancements in complex industrial comminution circuits.

Keywords: digital twin, grinding circuit optimisation, energy efficiency, first-principles modelling

INTRODUCTION

Digital transformation is progressively reshaping the mining industry, integrating advanced analytics, automation, and interconnected systems into mineral processing operations. Grinding circuits, encompassing equipment such as semi-autogenous grinding (SAG) mills, ball mills, and classification systems (hydrocyclones, screens), are a primary focus for these digitalisation efforts (Bascur & Soudek 2019; Young & Rogers 2019). As frequently the operational bottleneck of concentrators, the performance of the grinding circuit dictates overall plant throughput and significantly influences downstream recovery efficiency (Wills & Finch 2016). Even marginal improvements in mill throughput can yield substantial gains in productivity and profitability, making optimisation a critical objective (Giblett & Putland 2018). In a landscape marked by volatile commodity prices and stringent operational cost management, maximising the tons of ore processed per hour is critical to maintain competitiveness.

The energy consumption associated with comminution (crushing and grinding) further underscores the importance of optimisation. Grinding circuits are notoriously energy-intensive (Napier-Munn 2015), accounting for potentially 2–3% of global electricity consumption and often consuming up to 50% of a mine site's total energy budget (Allen Marc 2021). Consequently, enhancing grinding efficiency and throughput directly translates to reduced energy expenditure and lower unit operating costs.

Achieving even a small improvement in throughput or energy efficiency can lead to significant financial and resource conservation benefits.

Sustainability imperatives also drive the need for optimised grinding operations. Reducing the specific energy consumption (kWh per ton of ore processed) yields direct economical, and environmental benefits (Bouchard, Desbiens & Poulin 2017a). Digital optimisation tools enable mines to continuously adjust operating parameters to maintain peak performance, potentially reducing energy usage by 5–15% without asset modification (W.A. Gough 2015; Bouchard *et al.*, 2017a). This involves utilising existing processes more effectively, enhancing both energy efficiency and throughput concurrently. Furthermore, stable and efficient grinding circuit operation can minimise waste generation, improve downstream metallurgical recovery, and potentially extend the operational life of major equipment, contributing to more sustainable mining practices.

Grinding circuits present unique and significant operational challenges due to their inherent characteristics and their pivotal role in the overall mineral processing flowsheet. The primary goal, typically maximising throughput while achieving a target product size distribution (often characterised by d80, the particle size at which 80% of material cumulatively passes) suitable for downstream separation processes (e.g., flotation, leaching), is complicated by several factors:

- *Process Complexity and Non-Linearity:* Comminution involves intricate physical phenomena like particle fracture, material transport within mills, and classification in hydrocyclones or screens. These processes exhibit highly non-linear behaviour. For instance, mill power draw does not scale linearly with feed rate or mill speed, and hydrocyclone separation efficiency is a complex function of feed properties and operating pressure (Olivier & Aldrich 2021). The interactions between units are strong; changes in SAG mill operation directly impact the feed to the ball mill and hydrocyclones, creating coupled dynamics that are difficult to manage independently.

- *Variability:* Ore feed characteristics are rarely constant. Fluctuations in hardness, mineralogy, feed size distribution, and moisture content significantly affect grinding behaviour and energy requirements. Equipment conditions also vary due to wear. These variations act as continuous disturbances to the circuit (Sikström, Brown & Koorts 2022).

- *Time Delays:* Grinding circuits exhibit considerable transport and process lags. The response of the product size distribution to a change in feed rate or water addition may take tens of minutes to fully manifest, making real-time control tuning challenging (Yamashita *et al.*, 2023).

- *Measurement Limitations:* Key internal state variables, such as the precise particle size distribution within the mill, fill level, ore/ball charge or the circulating load composition, are often difficult or impossible to measure directly and continuously (Hodouin *et al.*, 2001). Operators rely on indirect measurements (e.g., mill acoustics and vibration monitoring, power draw, sump levels, density gauges) and infrequent laboratory samples, leading to incomplete process visibility.

- *High Energy Consumption and Grinding Efficiency:* Comminution is fundamentally energy-inefficient, with only a small fraction (~1–5%) of input energy creating new particle surfaces (Bouchard, Desbiens & Poulin 2017b). Inefficiencies arise from designs mismatched to current and drifting ore properties, high recirculating loads reprocessing already fine material, and operational deviations from optimal slurry density or mill load.

- *Throughput Maximisation Constraints:* Often, the SAG mill or the ball mill circuit limits overall tonnage. Relieving these bottlenecks might involve incremental upgrades like adding pebble crushing capacity, optimising classification, or improving feed preparation. However, each fix has limits, and pushing throughput can lead to trade-offs, such as increased wear rates, higher maintenance costs, or coarser product size potentially impacting downstream recovery. Determining the true economic optimum, balancing throughput gains against costs and recovery impacts, is complex due to these interacting constraints.

Traditional methods for tackling previous challenges have been through basic proportional-integral-derivative (PID) control loops for regulatory control, supplemented by manual oversight from experienced operators (Olivier & Aldrich 2021). However, the inherent complexity, non-linearity, significant time delays, and strong interdependencies within grinding circuits make purely manual or basic automated control suboptimal. Frequent variations in ore characteristics (e.g., hardness, mineralogy, feed particle size distribution (PSD)) and equipment condition continuously perturb the system, demanding constant adjustments that are difficult to optimise manually or with simple control logic (Ramasamy, Narayanan & Rao 2005). This often leads to conservative operation within a narrow 'comfort zone', sacrificing potential throughput and efficiency.

Advanced process control (APC) techniques, such as rule-based expert systems, fuzzy logic, and model predictive control (MPC), have been applied to grinding circuits with varying degrees of success. MPC leverages a dynamic process model to predict future behaviour and optimise multiple control actions simultaneously, while respecting constraints (Thivierge, Bouchard & Desbiens 2023). However, challenges remain, including the need for accurate and well-maintained process models, operator trust and acceptance, and the expertise required for implementation and tuning.

More recently, data-driven approaches using machine learning (ML) and artificial intelligence (AI) have emerged (Di Feo *et al*., 2024). Techniques like decision trees have been used to extract operational rules from historical data (Olivier & Aldrich 2021). Reinforcement learning (RL) has shown promise in simulation studies, learning complex control policies to optimise profit-based objectives (Hallén *et al*., 2019).

The proposed approach seeks to overcome the limitations of prior methods by utilising a first-principles based digital twin framework. This framework leverages fundamental engineering models of each unit operation, ensuring physical realism and adherence to mass and energy balances. Critically, these models are calibrated using plant-specific operational data, creating a hybrid approach that combines the predictive power of physics with site-specific empirical adjustments. The contribution of this work lies in demonstrating the development, calibration, and application of the proposed framework to an industrial gold processing circuit, showcasing significant performance improvements over an established baseline.

GRINDING CIRCUIT DIGITAL TWIN FRAMEWORK

The proposed framework provides a robust platform for simulating and optimising grinding circuit performance. It centres around a digital twin constructed from interconnected first-principles models of the individual unit operations, calibrated against plant data.

Overview

The digital twin serves as a high-fidelity virtual replica of the physical grinding circuit. It is designed to be modular, allowing individual unit models to be updated or replaced without affecting the overall structure. The core comminution model components simulate the physical processes occurring within each piece of equipment, while the digital twin framework manages the flow of material and information between these components. An optimisation layer sits atop the simulation, systematically adjusting decision variables to achieve a defined objective (e.g., maximise throughput), subject to process constraints.

The simulation proceeds sequentially following the comminution process flowsheet. The output streams (solid flow rate vector and water flow rate) from one unit model become the input streams for the next. For closed circuits, the recirculation estimators execute internal iterations, passing stream data between the relevant unit models until a steady-state mass balance is achieved. This ensures that the overall circuit simulation represents a converged state. The optimisation engine then orchestrates the main simulation modules with trial decision variable values. The simulation is executed (including loop solver iterations), calculates key performance indicators (KPIs) and constraint values, and returns the penalised objective function value to the optimiser, which suggests the next trial variables until an optimal threshold is met. The framework may interact with plant data sources like the distributed control system (DCS) or historians for calibration and potentially for online updates.

It is important to distinguish the timescale of the digital twin's calculated optimal setpoints from the underlying process control layers. While low-level PID controllers adjust actuators rapidly, often on a seconds timescale, the supervisory setpoints targeted by the optimisation (e.g., optimal mill feed rate, target slurry density, mill speed setpoint) are typically adjusted much less frequently. Due to the inherent process dynamics and response times of large grinding equipment, meaningful changes to these supervisory setpoints usually aim at steady-state optimisation and occur on the order of minutes, whether adjusted manually by operators or by higher-level control systems. The computational time required for the digital twin optimisation scenarios in this study aligns well with this minute-scale cadence. This confirms the framework's suitability for providing timely decision support in an offline advisory capacity or potentially serving as the calculation engine for an online supervisory control layer that updates targets for the plant's DCS at operationally relevant intervals.

Core Framework Components

The digital twin framework is built upon several interconnected components designed to simulate the grinding circuit and identify optimal operating conditions. The core components include:

Comminution Simulation Engine: The core of the framework, comprised of steady-state simulation models that mathematically describe the steady-state physical transformations (size reduction, separation) occurring within each unit. The modular engine is comprised of the following models; for a detailed breakdown of all model input, outputs and calibration parameters please refer to the Appendix A:

- ***Unit Operation Models:***
  - *SAG Mill model (SAGM):* Simulates ore breakage, internal transport, and grate discharge using population balance concepts. Key inputs include mill speed (ϕ), ball charge (JB), and feed PSD (di). Outputs include product PSD (P_pi), power draw (W), and internal charge volume (J). The formulation integrates breakage kinetics and discharge mechanisms akin to those defined by Hinde & Kalala (2009).
  - *Ball Mill model (BLM):* Utilises similar comminution principles for finer grinding, predicting product PSD (P_pi) and power draw (W) based on speed (ϕ), ball charge (JB), and feed (F_fi).
  - *Crusher model (CRM):* Models size reduction of coarse particles based on closed-side setting (Css). It uses classification and breakage functions derived from principles established in models like Whiten (1972).
  - *Hydrocyclone model (HCM):* Simulates particle classification based on cyclone geometry (Dc, Do, Du, etc.), operating pressure (P), and feed slurry properties. It employs the widely adopted semi-empirical framework developed by Flintoff, Plitt & Turak (1987).
  - *Screen (SCM) & Trommel (TRM) model:* simulates size separation based on aperture h, wire d, angle theta, geometry. Efficiency calculations are primarily adapted from the Karra (1979) screening model.
  - *Pump Box models:* Modelled implicitly through mass and water balance calculations at stream mixing points, summing incoming flows and incorporating direct water addition (Wa).
- **Recirculation Estimators (*SAGExM, HCExM*):** Essential modules within the simulation engine that handle closed-circuit mass balancing. They iteratively pass stream data (solids flow vectors, water flows) between interconnected unit models (e.g., SAG-

Crusher-Screen loop, Ball Mill-Hydrocyclone loop) until the recirculating flows converge to a steady state.

  - **KPI Estimators:** Calculate KPIs like particle size d80 and solids fraction (Xp) from the simulated stream data.

- **Optimisation Engine:** Leverages Bayesian Optimisation (BO) to systematically drive the simulation engine. It proposes trial values for decision variables, receives the resulting objective function value (hydro cyclone throughput), and intelligently selects subsequent trials to find the optimum.

- **Data Interface:** Connectors for managing ore feed characteristics (di, Fi), operational parameters, decision variable bounds, constraint bounds, and model parameters. Moreover, manages the integration of the identified optimal control setpoints with its proper destination.

While the unit operation models are grounded in first principles, they incorporate numerous parameters (e.g., breakage rate coefficients; classification parameters; power coefficients) that are determined through calibration against plant-specific operational data. This calibration step embeds site-specific empirical behaviour, making the overall digital twin a hybrid model, combining the predictive power and extrapolation capability of physics-based models with the accuracy derived from real-world plant performance.

Simulation Model Calibration
Calibration is a critical step to ensure the digital twin accurately reflects the actual plant behaviour. The methodology involved:

1. **Data Collection:** Gathering comprehensive operational survey data from the target processing plant. This includes feed characterisation, stream samples for PSD analysis throughout the circuit, flow rate measurements, density measurements (to calculate solids fraction Xp), mill power draws ($\dot{W}$), hydrocyclone pressures (P), and estimates of mill charge levels (J, JB).

2. **Parameter Estimation:** Adjusting tunable parameters within the first-principles models (e.g., breakage rate parameters S1E, K1, K2; classification parameters alpha, beta, F1-F4; power model coefficients) to minimise the discrepancy between model predictions and the measured survey data. This is typically an optimisation problem itself, often solved using least-squares or similar fitting techniques.

3. **Validation:** Comparing the predictions of the calibrated model against the survey data points for key KPIs:

   - Mass flow rates of solids and water in major streams.
   - PSDs of key streams (e.g., mill discharge, cyclone overflow/underflow, screen oversize/undersize).
   - Hydrocyclone operating pressure (P).
   - SAG and Ball Mill power draws ($\dot{W}$).
   - SAG and Ball Mill volumetric charge fractions (J).

Optimisation Model Formulation
The calibrated simulation model is embedded within an optimisation framework to identify the best operating conditions.

- **Optimisation Algorithm:** The optimisation was conducted using BO, a method well-suited for computationally expensive, black-box functions like grinding circuit simulations. BO constructs a probabilistic surrogate model of the objective function, updating it after each simulation run to learn the landscape of the decision variable space. An acquisition function then guides the selection of the next evaluation point by balancing exploitation (sampling where predicted values are high) and exploration (sampling where uncertainty is greatest). After an initial phase of random sampling to seed the surrogate model, the process iteratively selects the point that maximises the acquisition function, runs the simulation, and updates the model. This approach significantly reduces the number of costly evaluations required to reach near-optimal solutions, outperforming traditional methods like grid or random search.

- **Objective Function:** The primary objective is to maximise the hydrocyclone overflow solids throughput (which is equivalent to the plant feed rate at steady-state). As implemented, a penalty term related to constraint violations and the recirculation estimators convergence error is included to ensure stable simulation results.

- **Decision Variables (Inputs):** These are the adjustable parameters manipulated by the optimisation algorithm to find the optimal operating point. They encompass both short-term operational controls and longer-term configuration settings, allowing for a multi-faceted optimisation approach:

    - Short-term Control Variables: Plant feed rate (SAG_F), water addition rates at various points (the SAG feed chute and both pump boxes), and SAG mill rotational speed as a fraction of critical speed (SAG_phi).

    - Medium/Longer-term Configuration Variables: Volumetric fraction occupied by balls in SAG and Ball mills (SAG_JB, BM_JB), hydrocyclone diameters (vortex HC_Do, apex HC_Du), screen design parameters (SC_Gw, SC_h, SC_d, SC_theta), crusher closed-side setting (CR_CSS), Ball mill fraction of critical speed (BM_phi). The specific set of variables included in the optimisation depends on the nature of the optimisation scenario being evaluated, for example control variable setpoint optimisation, or what-if scenario configuration evaluation. Each decision variable is constrained within predefined lower (LB) and upper (UB) bounds reflecting physical or operational limits.

- **Constraints:** Operational and equipment limitations were enforced as constraints using penalty functions added to the objective function value. Penalty factors were tuned heuristically to strongly discourage constraint violations without causing numerical instability during the search. The optimisation ensured adherence to the following key user-specified upper and lower bounds:

    - **Mill Constraints:**
        - SAG Mill Solids Fraction: Maintain appropriate slurry density for transport/trommel
        - SAG Mill Power Draw: Prevent motor overload / inefficient operation
        - SAG Mill Total Charge Volume: Avoid mill overloading or underutilisation which would increase liner wear
        - Ball Mill Power Draw: Prevent motor overload / inefficient operation

    - **Hydrocyclone Constraints:**
        - Operating Pressure: Ensure stable classification and prevent roping/short-circuiting
        - Overflow d80 (Product Quality): Meet downstream process requirements
        - Underflow Solids Fraction: Ensure suitable feed density for the Ball Mill

- **Pump Box Constraints:**
  - Pump Box Solids Fraction: Maintain pumpable slurry density for screen feed

Framework Capabilities and Implementation Considerations

The first-principles digital twin framework presented offers holistic optimisation, capturing complex interactions between unit operations and identifying optimal strategies at the interface of various units, rather than local adjustments common with unit-level control. A key strength lies in its inherent ability to handle constraints; physical limits (e.g., mass/energy balances) are naturally embedded, and operational constraints (e.g., power limits, pressure ranges, product size targets) are incorporated into the optimisation, ensuring recommendations are executable. Furthermore, the modular structure enhances flexibility, allowing the framework to be adapted to different circuit configurations or extended to include upstream (e.g., blasting, blending) or downstream (e.g., flotation) process models.

Realising these benefits, however, necessitates careful consideration of certain implementation aspects. The fidelity and predictive accuracy of the digital twin is highly dependent on the availability of comprehensive, high-quality plant survey data for thorough initial calibration. Moreover, maintaining this accuracy over time requires a commitment to periodic model maintenance and recalibration to account for factors like evolving ore hardness characteristics or gradual equipment wear. Therefore, while the framework provides powerful analytical and optimisation capabilities, its successful deployment hinges on adequate resources for model tuning, robust data acquisition practices, and ongoing model stewardship.

CASE STUDY

The developed digital twin framework was applied to optimise the grinding circuit of a gold processing facility located in West Africa.

Background and Scope

The facility processes gold ore from underground operations, with the comminution circuit, shown in Figure 1, playing a key role in liberating gold before leaching. The circuit includes typical components such as a SAG mill, crusher, vibrating screen, ball mill, and hydrocyclones arranged in a closed-loop configuration. The primary operational objective was to maximise the throughput of solids in the hydrocyclone overflow stream while maintaining product quality and respecting all equipment and process constraints.

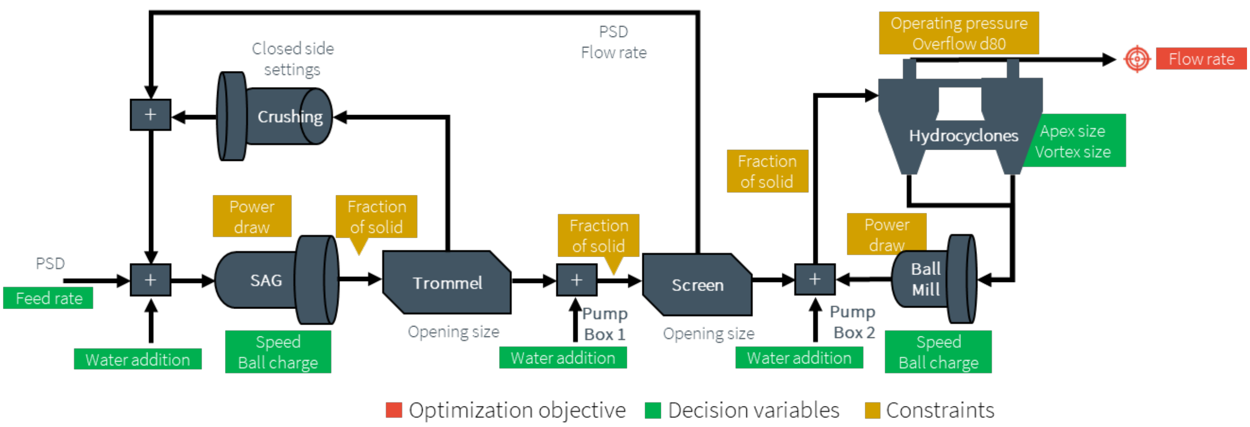

Figure 1. Flowsheet diagram representing the process under which the proposed framework is implemented. Detailing decision variables in green, objectives in red and constraints in orange.

The scope involved developing and calibrating the first-principles digital twin for the comminution circuit described above and subsequently using it within an optimisation framework to identify optimal operational setpoints and assess the impact of potential configuration changes.

The solution utilised the Python-based digital twin framework detailed in Section 2. This included the calibrated unit operation models (SAGM, BLM, TRM, CRM, SCM, HCM), recirculation estimators, KPI calculators (d80M, XPM), and the Bayesian optimisation engine. The framework operated offline, taking current conditions and constraints as input and providing recommended optimal setpoints.

Evaluation KPIs
The primary KPI for optimisation was the hydrocyclone overflow solids mass flow rate (HC_TTPo, t/h). Key performance indicators used for monitoring and constraint checking included:

- Hydrocyclone overflow particle size d80 (HC_do80, mm)
- SAG mill power draw (SAG_Ẇ, kW)
- Ball mill power draw (BM_Ẇ, kW)
- Hydrocyclone operating pressure (HC_P, kPa)
- SAG mill volumetric fill level (SAG_J, %)
- Solids fraction in various streams (PB1_Xp, SAG_Xp, HC_Xu, %)

Model Calibration Approach
The digital twin models were calibrated using detailed survey data collected on February 29th, 2024. This involved tuning model parameters within the process units modules to match measured values of stream flow rates, PSDs (at multiple points in the circuit), mill power draws, hydrocyclone pressure, and solids fractions. Validation plots comparing model predictions against measured data confirmed a generally good fit across the circuit (see Figure 2), providing confidence in the model's predictive accuracy for the baseline conditions.

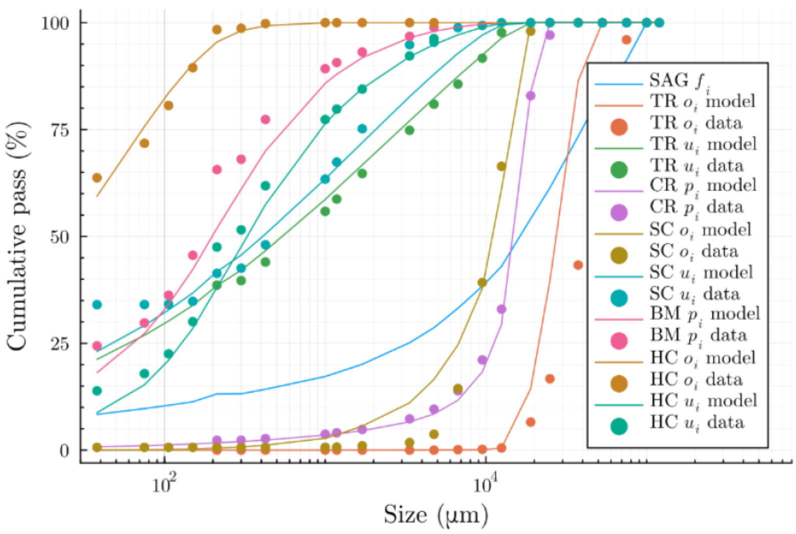

Figure 2. Calibration curve of developed simulation models against the industrial dataset, highlighting model fit to the sampled points.

Some minor discrepancies were observed for the trommel oversize PSD (*TR o*, low flow rate stream), but this is not worrying because its flow rate is negligible at the current operating conditions (53 t/h, relative to 500-1500 t/h in the other streams). Some abnormal behaviour was also observed for the finest fractions of the screen undersize, potentially due to sampling/assay error, but it remains small enough to not significantly impact the overall optimisation conclusions.

Results

A reference base case was established using operating conditions aligned with the calibration survey data. This base case operated with a SAG mill feed rate (*SAG_F*) of 410 t/h, 13 hydrocyclones in parallel (*HC_ncp*=13), a SAG mill speed (*SAG_phi*) of 73.3% critical, and SAG/Ball mill ball charges (SAG_JB, BM_JB) of 15% and 34% respectively. This resulted in a hydrocyclone overflow throughput (HC_O_oi) of 409 t/h, with a corresponding product size (HC_do80) of 102 µm. Key process variables included SAG power draw (SAG_Ẇ) of 1351 kW, ball mill power draw (BM_Ẇ) of 3146 kW, and hydrocyclone pressure (HC_P) of 125 kPa.

Scenario evaluation was executed on a workstation (Intel Core i7-10850H @ 2.70GHz, 32GB RAM), a single simulation run of the entire circuit completed in an average of 6 seconds. A full optimisation scenario involving multiple iterations typically completed in an average of 251 seconds, demonstrating practicality for offline daily decision support and infrequent supervisory setpoint optimisation. Three optimisation scenarios were then executed using the calibrated digital twin framework, with a summary of results highlighted in Table I:

- *Scenario 1:* Control Variables Optimisation: The optimiser adjusted only short-term control variables (*SAG_F, SAG_Wa, SAG_phi, PB1_Wa, PB2_Wa*), keeping equipment configuration fixed (*HC_ncp=13, fixed SAG_JB, BM_JB, etc.*).
    - *Result:* Achieved a hydrocyclone overflow throughput of 423 t/h, representing a 3.4% increase over the base case. This was primarily achieved by increasing the SAG mill speed (SAG_phi) significantly to 79% of critical and adjusting the water balance (notably higher SAG_Wa). Consequently, SAG power draw (SAG_Ẇ) rose to 1526 kW, and hydrocyclone pressure (HC_P) increased to 139 kPa (approaching the upper operating limit). The product size (HC_do80) slightly coarsened to 98 µm, remaining within an acceptable range relative to the typical target.
- *Scenario 2:* Control and Configuration Variables Optimisation: The optimiser adjusted both short-term control variables and key medium-term configuration parameters (SAG_JB, BM_JB), while CR_CSS remained fixed in this run. The number of hydrocyclones remained at 13.
    - *Result:* Achieved a hydrocyclone overflow throughput of 441 t/h, a 7.8% increase over the base case (a further ~4.4% gain over Scenario 1). This improvement was driven by maximising the SAG mill speed (SAG_phi to 80%), and increasing the ball charges in both mills (SAG_JB to 17%, BM_JB to 40%). These changes led to higher power draws (SAG_Ẇ at 1609 kW, BM_Ẇ at 3354 kW), while hydrocyclone pressure remained high (142 kPa) and product size (HC_do80) stayed at 98 µm. This scenario demonstrates the benefit of adjusting ball loads to support higher tonnage.
- *Scenario 3:* Additional Hydrocyclone and Full Optimisation: This scenario explored a minor design modification by enabling an additional (14th) hydrocyclone (HC_ncp=14), while allowing optimisation across control and configuration variables (including water addition points PB1_Wa, PB2_Wa).
    - *Result:* Achieved a hydrocyclone overflow throughput of 472 t/h, a substantial 15.4% increase over the base case (a further ~7.6% gain over Scenario 2). The key enabler was the increased classification capacity from the fourteenth hydrocyclone, which increased the ball mill circuit capacity . The optimisation maintained high SAG speed (80%) and high ball charges (17% / 40%). Interestingly, the SAG power draw (1571 kW) was slightly lower than Scenario 2 despite higher feed, likely due to optimised water balance (activating PB1_Wa, adjusting SAG_Wa and PB2_Wa). Ball mill power remained high (3349 kW). Critically, the hydrocyclone pressure (142 kPa) was

maintained within limits despite the significantly higher total flow, as the flow per cyclone was managed by the additional unit. The product size (HC_do80) improved back to 101 µm, comfortably meeting the quality target, and the hydrocyclone underflow density (HC_Xu) reached its upper limit of 84%.

Table I. Summary of Optimisation Scenario Results

Scenario	Optimised Variables	Throughput Increase (vs. Base)	Key Constraint Met (Example: HC d80 ≤ 102 µm)
Reference Base Case	Fixed at survey conditions	-	102
#1: Control Optimisation	SAG_F, SAG_Wa, SAG_phi, PB1_Wa, PB2_Wa (13 HCs, fixed config)	3.0%	98
#2: Control & Config Optimisation	Scenario 1 variables + SAG_JB, BM_JB	7.6%	98
#3: Control, Config & +1 Hydrocyclone	Scenario 2 variables + with HC_ncp=14	15.0%	101

Significant throughput improvements were identified, progressing from operational tuning to configuration adjustments and minor design modifications. While this paper's primary objective was throughput maximisation, the framework could readily be adapted to co-optimise or constrain specific energy consumption (kWh/t). Quantification of energy efficiency impacts represents an area for future analysis. These findings confirm the readiness of the digital twin for operational use as an offline advisory tool and provide a strong foundation for transitioning to near real-time deployment.

CONCLUSIONS

Advancements in digital technology are occurring rapidly. We have demonstrated the application of a comprehensive grinding circuit digital twin framework, grounded in first-principles modelling, and BO for the operational improvement at a gold processing facility. By integrating detailed physics-based models of individual unit operations and calibrating them against plant survey data, a high-fidelity virtual replica of the circuit was established. The hybrid nature of the models provided a robust and flexible foundation for simulating and optimising process behaviour in response to variable ore feed PSD and equipment constraints.

The results from three distinct optimisation scenarios were observed, starting with a 3% throughput gain achieved solely by optimising short-term control variables. Incorporating medium-term configuration variables (like mill ball charges) yielded a cumulative gain of 7.6%. The most substantial gain, a 15% increase in throughput relative to the base case, was realised when both operational and configuration variables were optimised alongside a structural change (enabling an additional hydrocyclone). Importantly, across all scenarios, the optimisation respected all specified process and equipment constraints, reinforcing the viability of the recommendations for potential real-world implementation.

Beyond the quantified throughput gains, this paper provides a replicable and technically sound framework for the mining industry. It enables scenario-based planning, allows for the quantitative assessment of the potential economic value of operational changes or design modifications (like adding classification capacity), and supports proactive, data-driven decision-making. The modular architecture of the digital twin allows for relatively seamless extension to include additional processing units or upstream/downstream integration (e.g., incorporating ore blending strategies or linking to flotation performance models). Its inherent simulation capability supports extensive 'what-if' analysis, helping engineers anticipate the impact of ore changes, mitigate potential production risks, and identify future bottlenecks before they materialise. The modular nature of the framework also makes it well-suited for integration with other KPIs, enabling future studies focused on specific energy consumption, or water usage.

Future work will focus on transitioning this offline decision support tool towards an online advisory system integrated with the plant's DCS and potentially leveraging data historians like the PI System. Key challenges for this transition include ensuring real-time data streaming capabilities and establishing robust fail-safe mechanisms and clear operator interface protocols.

APPENDIX A

Abbreviations

Acronym	Definition
AI	*Artificial Intelligence*
APC	*Advanced Process Control*
BM	*Ball Mill*
CR	*Crusher*
CSS	*Closed-Side Setting*
d80	*Particle diameter at which 80% of material cumulatively passes*
DCS	*Distributed Control System*
HC	*Hydrocyclone*
JB	*Volumetric fraction occupied by balls (charge)*
KPI	*Key Performance Indicator*
ML	*Machine Learning*
MPC	*Model Predictive Control*
PB	*Pump Box*
PID	*Proportional-Integral-Derivative*
PIML	*Physics-Informed Machine Learning*
PSD	*Particle Size Distribution*
RL	*Reinforcement Learning*
SAG	*Semi-Autogenous Grinding*
SC	*Screen*

TR	*Trommel*
ϕ (phi)	*Mill rotational speed as fraction of critical speed*

Detailed Model Inputs & Outputs

Table 2: SAG Mill Model (SAGM) Parameters and Variables

Variable Name	Symbol	Category	Unit	Definition/Description
Inputs				
Ore Feed Size Classes	*di*	Calibrated Input/Characterisation	mm	Vector defining discrete particle size intervals.
Ore Feed Mass Flow Rate	*SAG_F*	User-Specified (Scenario Input)	t/h	Total solids mass flow rate fed to the circuit (primary feed).
Feed Ore PSD Fraction	*F_i*	Calibrated Input/Characterszation	-	Fraction of total feed solids (SAG_F) within each size class di.
Recirculating Solids Feed PSD	*G_gi*	Internal (from Loop Solver)	t/h	Solids mass flow rate from crusher/screen recycle for each size class di.
Feed Water Mass Flow Rate	*Wf*	Calculated Internally	t/h	Total water entering SAG (from fresh feed moisture + recycle + addition).
Added Water Mass Flow Rate	*SAG_Wa*	User-Specified (Scenario Input)	t/h	Water added directly to the SAG mill feed.
Recirculating Water Feed	*Wg*	Internal (from Loop Solver)	t/h	Water mass flow rate from crusher/screen recycle.
Fraction of Critical Speed	*SAG_phi (ϕ)*	User-Specified (Scenario Input)	-	Mill rotational speed as a fraction of its critical speed.
Volumetric Fraction of Balls	*SAG_JB*	User-Specified (Scenario Input)	-	Volume occupied by grinding balls

				relative to total mill volume.
Ore Solid Density	*rhos (ρs)*	Calibrated Input/Characterisation	t/m³	Average density of the solid ore particles.
Steel Ball Density	*rhoB (ρB)*	Static/Design	t/m³	Density of the grinding media (steel balls).
Mill Geometry (Diameter, Lengths, Trunnion)	*dm, L, Lc, dt*	Static/Design	m	Internal dimensions of the SAG mill shell.
Breakage Rate Parameters	*a1,a2,K1,K2,μ,λ*	Calibrated	various	Coefficients defining the energy-normalised breakage rate function (siE).
Discharge Classification Parameters	*d50c, m*	Calibrated	mm, -	Parameters defining the grate/discharge classification function (ki).
Discharge Model Parameters	*kprime,m2,dg,γ,A*	Calibrated	various	Parameters for the Napier-Munn discharge flow model.
Charge Porosity	*epsilon (ϵ)*	Calibrated/Assumed	-	Effective void fraction within the mill charge.
Power Draw Model Coefficient	*KWd (K_W)*	Calibrated	kW/(t√m)	Coefficient relating mill size, load, and speed to power draw.
Euler Integration Parameters	*TOL_EULER, DELTA_T_EULER, MAXITER_EULER, AASYMP*	Static/Solver Setting	-	Parameters controlling the numerical solver for mill holdup.
		Outputs		
Product Solids Mass Flow Rate per Size Class	*P_pi*	Output	t/h	Solids mass flow rate exiting the mill for each size class di.

Product Water Mass Flow Rate	*Wp*	Output	t/h	Water mass flow rate exiting the mill.
Volumetric Fraction of Total Charge	*J*	Output	-	Volume occupied by total charge (ore+balls+slurry) relative to mill vol.
Mill Power Draw	*Ẇ (W)*	Output	kW	Calculated electrical power consumed by the mill motor.
Holdup Convergence Error	*dhx*	Output (Solver Diagnostic)	various	Residual error from the Euler integration for steady-state holdup.

Table 3. Ball Mill Model Parameters and Variables

Parameter/Variable Name	**Symbol**	**Category**	**Unit**	**Definition/Description**
		Inputs		
Ore Feed Size Classes	*di*	*Calibrated Input/Characterisation*	*mm*	*Vector defining discrete particle size intervals.*
Solids Feed PSD	*F_fi*	*Internal (from Hydrocyclone U/F)*	*t/h*	*Solids mass flow rate from hydrocyclone underflow for each size class di.*
Water Feed Mass Flow Rate	*Wf*	*Internal (from Hydrocyclone U/F)*	*t/h*	*Water mass flow rate from hydrocyclone underflow.*
Fraction of Critical Speed	*BM_phi (ϕ)*	*User-Specified (Scenario Input)*	*-*	*Mill rotational speed as a fraction of its critical speed.*
Volumetric Fraction of Balls	*BM_JB*	*User-Specified (Scenario Input)*	*-*	*Volume occupied by grinding balls relative to total mill volume.*
Ore Solid Density	*rhos (ρs)*	*Calibrated Input/Characterisation*	*t/m³*	*Average density of the solid ore particles.*
Steel Ball Density	*rhoB (ρB)*	*Static/Design*	*t/m³*	*Density of the grinding media (steel balls).*
Mill Geometry (Diameter, Lengths, Trunnion)	*dm,L,Lc,dt*	*Static/Design*	*m*	*Internal dimensions of the ball mill shell.*

Energy-Normalised Breakage Rate Params	*S1E, dref, zeta1 (ζ1)*	*Calibrated*	*various*	*Coefficients defining the energy-normalised breakage rate function (siE).*
Charge Porosity	*epsilon (ϵ)*	*Calibrated/Assumed*	-	*Effective void fraction within the mill charge.*
Power Draw Model Coefficient	*KWd (K_W)*	*Calibrated*	*kW/(t√m)*	*Coefficient relating mill size, load, and speed to power draw.*
		Outputs		
Product Solids Mass Flow Rate per Size Class	*P_pi*	*Output*	*t/h*	*Solids mass flow rate exiting the mill for each size class di.*
Product Water Mass Flow Rate	*Wp*	*Output*	*t/h*	*Water mass flow rate exiting the mill.*
Mill Power Draw	*Ẇ (W)*	*Output*	*kW*	*Calculated electrical power consumed by the mill motor.*
Volumetric Fraction of Total Charge	*J*	*Output*	-	*Volume occupied by total charge (ore+balls+slurry) relative to mill vol.*

Table 4. Hydrocyclone Model Parameters and Variables

Parameter/Variable Name	Symbol	Category	Unit	Definition/Description
		Inputs		
Ore Feed Size Classes	*di*	*Calibrated Input/Characterisation*	*mm*	*Vector defining discrete particle size intervals.*
Solids Feed PSD	*F_fi*	*Internal (from Pump Box 2)*	*t/h*	*Solids mass flow rate from PB2 for each size class di.*
Water Feed Mass Flow Rate	*Wf*	*Internal (from Pump Box 2)*	*t/h*	*Water mass flow rate from PB2.*
Number of Cyclones in Parallel	*HC_ncp*	*Static/Design or Scenario Input*	-	*Number of identical hydrocyclones operating.*
Overflow Nozzle Diameter (Vortex Finder)	*HC_Do*	*User-Specified (Scenario Input)*	*cm*	*Diameter of the overflow outlet.*
Underflow Nozzle Diameter (Apex)	*HC_Du*	*User-Specified (Scenario Input)*	*cm*	*Diameter of the underflow outlet.*

Ore Solid Density	*rhos (ρs)*	*Calibrated Input/Characterisation*	*t/m³*	*Average density of the solid ore particles.*
Cyclone Diameter	*Dc*	*Static/Design*	*cm*	*Main cylindrical diameter of the hydrocyclone body.*
Free Vortex Height	*h*	*Static/Design*	*cm*	*Height of the free vortex zone within the cyclone.*
Feed Nozzle Diameter	*Di*	*Static/Design*	*cm*	*Diameter of the tangential feed inlet.*
Liquid Density	*rhol (ρl)*	*Static/Assumed*	*t/m³*	*Density of the carrier fluid (water).*
Liquid Viscosity	*eta (η)*	*Static/Assumed*	*cP*	*Viscosity of the carrier fluid (water) at a reference temperature.*
Plitt Model Calibration Parameters	*F1, F2, F3, F4*	*Calibrated*	-	*Empirical coefficients for the Plitt hydrocyclone model equations.*
Outputs				
Overflow Solids Mass Flow Rate per Size Class	*O_oi*	*Output*	*t/h*	*Solids mass flow rate exiting via overflow for each size class di.*
Underflow Solids Mass Flow Rate per Size Class	*U_ui*	*Output*	*t/h*	*Solids mass flow rate exiting via underflow for each size class di.*
Overflow Water Mass Flow Rate	*Wo*	*Output*	*t/h*	*Water mass flow rate exiting via overflow.*
Underflow Water Mass Flow Rate	*Wu*	*Output*	*t/h*	*Water mass flow rate exiting via underflow.*
Operating Pressure Drop	*P*	*Output*	*kPa*	*Pressure difference between feed and overflow/underflow.*
Parameter S (Related to Water Split)	*S*	*Output (Intermediate Calc)*	-	*Intermediate parameter in Plitt model related to water split to underflow.*

Table 5. Crusher Model Parameters and Variables

Parameter/Variable Name	Symbol	Category	Unit	Definition/Description
Inputs				
Ore Feed Size Classes	*di*	*Calibrated Input/Characterisation*	*mm*	*Vector defining discrete particle size intervals.*
Total Solids Feed Rate	*F*	*Internal (from Trommel O/S)*	*t/h*	*Total solids mass flow rate fed to the crusher.*
Solids Feed PSD Fraction	*fi*	*Internal (from Trommel O/S)*	-	*Fraction of total feed solids (F) within each size class di.*
Solids Fraction in Feed	*Xf (χf)*	*Internal (from Trommel O/S)*	-	*Mass fraction of solids in the crusher feed stream.*
Closed Side Setting	*CR_CSS*	*User-Specified (Scenario Input)*	*mm*	*Minimum discharge opening of the crusher.*
Classification Function Parameters	*alpha (α), beta (β), gamma (γ), k3*	*Calibrated*	*mm, mm, mm, -*	*Parameters defining the probability of a particle passing the CSS.*
Breakage Function Parameters	*K, m, nc (n)*	*Calibrated*	-, -, -	*Parameters defining the distribution of progeny particles after breakage.*
Outputs				
Product Solids Mass Flow Rate per Size Class	*P_pi*	*Output*	*t/h*	*Solids mass flow rate exiting the crusher for each size class di.*
Product Water Mass Flow Rate	*Wp*	*Output*	*t/h*	*Water mass flow rate exiting the crusher (assumed same as feed water).*

Table 6. Screen & Trommel Model Parameters and Variables

Parameter/Variable Name	Symbol	Category	Unit	Definition/Description
Inputs				
Ore Feed Size Classes	*di*	*Calibrated Input/Characterisation*	*mm*	*Vector defining discrete particle size intervals.*
Solids Feed PSD	*F_fi*	*Internal (from Pump Box 1)*	*t/h*	*Solids mass flow rate from PB1 for each size class di.*
Water Feed Mass Flow Rate	*Wf*	*Internal (from Pump Box 1)*	*t/h*	*Water mass flow rate from PB1.*
Water Addition Gain (Spray Water)	*SC_Gw*	*User-Specified (Scenario Input)*	*t/h per t/h solid*	*Rate of spray water added per tonne of solids feed (Karra model input).*
Screen Aperture Opening	*SC_h*	*User-Specified (Scenario Input)*	*mm*	*Nominal opening size of the screen mesh.*
Wire Diameter	*SC_d*	*User-Specified (Scenario Input)*	*mm*	*Diameter of the screen wires.*
Screen Deck Inclination Angle	*SC_theta (θ)*	*User-Specified (Scenario Input)*	°	*Angle of the screen deck relative to horizontal.*
Number of Screens in Parallel	*SC_ncp*	*Static/Design*	-	*Number of identical screens operating.*
Ore Solid Density	*rhos (ρs)*	*Calibrated Input/Characterisation*	*t/m³*	*Average density of the solid ore particles.*
Karra Model/Classification Parameters	*alpha (a), beta (β), a1,b1,c1,a2,c2*	*Calibrated*	*various*	*Empirical parameters for the Karra screening efficiency model.*
Oversize Solids Fraction	*Xo (χo)*	*Calibrated/Assumed*	-	*Target/Assumed solids mass fraction in the oversize stream (for water split).*
Deck Dimensions	*L1, L2*	*Static/Design*	*m*	*Width and length of the screen deck (used to calculate area sqm).*
Outputs				

Oversize Solids Mass Flow Rate per Size Class	*O_oi*	*Output*	*t/h*	*Solids mass flow rate exiting via oversize for each size class di.*
Undersize Solids Mass Flow Rate per Size Class	*U_ui*	*Output*	*t/h*	*Solids mass flow rate exiting via undersize for each size class di.*
Oversize Water Mass Flow Rate	*Wo*	*Output*	*t/h*	*Water mass flow rate exiting via oversize.*
Undersize Water Mass Flow Rate	*Wu*	*Output*	*t/h*	*Water mass flow rate exiting via undersize.*

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